Trial and Improvement

When faced with an equation we don't know how to solve, it is often good to guess. So given the equation $x^2 + x = 25$ then we create a table of trial values of x, $x^2 + x$ and a 'comment' column. The procedure is as follows:

- Write down equation to be solved.
- Write down the accuracy (number of decimal places) you wish to find in x. In this example we will use 2 decimal places.
- Guess a first value (usually a whole number) of x.
- Calculate $x^2 + x$ with your guess for x.
- In the comment column state whether it is too big or small.

And repeat this, each time 'sandwiching' the number ever tighter, moving in one decimal place at a time. So we start by finding x between two whole numbers (e.g. 4 and 5). Then finding x between two numbers one tenth apart (e.g. 4.5 and 4.6). And so on.
So the table should look as follows.

x	$x^2 + x$	Comment
4	20	Too small
5	30	Too big
4.5	24.75	Too small
4.6	25.76	Too big
4.53	25.0509	Too big
4.52	24.9504	Too small

Now we are asked to find x to 2 decimal places, and a lot of people would now write $x = 4.52$ (to 2 d.p.) **but this would be incorrect** and lose you marks in any test or exam you were taking. We are *correct* to say that the solution lies between 4.52 and 4.53 but we *cannot* yet say whether the answer is 4.52 or 4.53 to 2 d.p.

If it is closer to 4.52 then it would be 4.52 to 2 d.p. but if it is closer to 4.53 then it would be 4.53 to 2 d.p.

So I therefore must add another row to my table to test the point mid way between 4.52 and 4.53. Therefore I must test 4.525, giving

x	$x^2 + x$	Comment
4	20	Too small
5	30	Too big
4.5	24.75	Too small
4.6	25.76	Too big
4.53	25.0509	Too big
4.52	24.9504	Too small
4.525	25.000625	Too big

Now I can see that the solution lies between 4.52 and 4.525. All numbers in between these two are 4.52 to 2 d.p. and so I can now confidently express my answer as

$$x = 4.52 \text{ to } 2 \text{ d.p.}$$